

Henri Poincaré and the Principle of Relativity

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A review is made of the historical background of the Special Theory of Relativity with particular attention to the theoretical contributions of Henri Poincaré. The primary aims are to show the synthetic aspects of Einstein's original paper on relativity and to clarify the logic of his fundamental innovations.

INTRODUCTION

IN most elementary presentations of the Special Theory of Relativity (and in a number of advanced treatises as well) the historical background and evolution of the theory are discussed more or less cursorily, with the result that the reader may be left with a number of misconceptions about the respects in which Einstein's own achievement was unique. For example, in many accounts one is given the impression that Einstein was the first to transform mechanical relativity into a universal physical principle valid for all phenomena: mechanical, optical, and electrical. It would probably come as a surprise to most students to learn that this principle (with its extended meaning) had been proposed a decade before Einstein's original paper "On the Electrodynamics of Moving Bodies" (1905).¹ Similarly it is often implied that Einstein was the first to introduce an operational definition of time based on light signals, to recognize the relativity of simultaneity (with respect to time so defined) and thereby to show how the principle of relativity could be reconciled with the constancy of the velocity of light. All these impressions insofar as they are not unqualifiedly true are apt to create pedagogical difficulties, for it is much easier to understand Einstein's accomplishment if one sees how he was able to create an original synthesis containing a number of ideas which had already been introduced by other mathematical physicists of his time in their own efforts to create a satisfactory theory of electrodynamics.

While most writers on the subject tend to overstate the originality of various parts of

Einstein's theory, a minority represented by E. Whittaker takes the opposite position, contending that Einstein's role in the development of what Whittaker calls the "Relativity Theory of Poincaré and Lorentz" was a secondary one essentially limited to elaborating on the theoretical innovations of others, with few substantial additions of his own.² We believe that this view also fails to do justice to the available historical evidence and that it may also create obstacles for the student. For to appreciate the significance of Einstein's 1905 paper it is important to see how he was able to find a new point of view from which the whole theory of electrodynamics could be greatly simplified. As will be shown, Einstein played a unique role in establishing the universal validity of the principle of relativity and in revealing and capitalizing on its radical implications.

In this connection it is particularly instructive to examine certain articles on mathematical physics published by Henri Poincaré during the period 1895–1905. For of all Einstein's contemporaries Poincaré seems now to have come closest to anticipating him in respect to the Special Theory. One of the subsidiary aims of this paper will be to suggest some of the preconceptions and reservations that may have held him back.

FORMULATION OF THE PRINCIPLE OF RELATIVITY

The first stage in the evolution of the Special Theory of Relativity is generally recognized as beginning with the failure to detect experimentally the motion of the earth through the ether. Although the details of these experiments and the theoretical considerations which made

¹ A. Einstein, *Ann. Physik* **17**, 891 (1905) as reprinted in H. A. Lorentz, A. Einstein, *et al.*, *The Principle of Relativity*, translated by W. Perrett and G. B. Jeffery (Methuen and Company Ltd., London, 1923), p. 37.

² Sir Edmund Whittaker, *A History of the Theories of Aether and Electricity: The Modern Theories 1900–1926* (Philosophical Library, New York, 1954), p. 40.

them seem practicable are familiar, we shall review the situation briefly to provide the background for a very important theoretical innovation made by Poincaré. Contemporary electrodynamics had been able to account satisfactorily for the absence of first-order (v/c) effects resulting from the absolute motion of the earth (i.e., through the ether). But there had been left open the possibility of detecting second-order (v^2/c^2) effects provided one could construct experimental apparatus of sufficient sensitivity. For this purpose A. A. Michelson's interferometer was improved to a point where in an experiment conducted by him in 1887 in collaboration with E. W. Morley it seemed assured that the predicted effects of the earth's motion could be discerned. When the results of this experiment were also negative, a serious theoretical problem arose, and physicists were unusually challenged in their search for an explanation. The one which seemed most promising, although radical, was the hypothesis proposed by H. A. Lorentz and independently by G. F. Fitzgerald that the expected results of the earth's motion had been offset by another effect of the motion on the experimental apparatus,³ namely a contraction of the arm of the interferometer in the direction of the motion by the factor $(1 - v^2/c^2)^{1/2}$. This hypothesis, in turn, left open the possibility of designing some new experiment to detect the earth's motion, which would not be aborted by the contraction effect or which might in some manner capitalize on the contraction itself, and in fact a number of such experiments were later conducted, although none was successful.

As early as 1895 Poincaré had expressed his dissatisfaction with the contemporary approach to the foregoing experiments.⁴ On philosophical grounds he objected to the number of separate hypotheses Lorentz and others were introducing to account for the failure of each experiment.⁵

³ See H. A. Lorentz, *Versuch einer Theorie der electrischen und optischen Erscheinungen in bewegten Körpern* (E. J. Brill, Leiden, 1895), pp. 120–125. On p. 122, footnote 2, Lorentz cites the earliest appearances of this hypothesis. In connection with the importance of the Michelson–Morley experiment in the development of Einstein's theory one should note that Einstein was familiar with this work by Lorentz. See G. Holton, "On the Origins of the Special Theory of Relativity," *Am. J. Phys.* 28, 635, footnote reference 33 (1960), for evidences of Einstein's familiarity with Lorentz's work of 1895.

⁴ H. Poincaré, *L'Éclairage Électrique* 5 (#40), 14 (1895).

⁵ H. Poincaré, *Rapports présentés au Congrès Inter-*

For example, it seemed to him unsatisfactory that different hypotheses had been introduced to explain the absence of first and second order effects, respectively. According to him this required a very unlikely *coup de pousse* or "rigging of the scales" on nature's part in each case,⁶ and he was emphatic in urging that one should adopt a more general point of view.

Actually, Poincaré had already convinced himself of the impossibility of detecting the absolute motion of the earth by any experimental means: mechanical, optical or electrical; and he was ready to conclude that all experiments with that objective would prove abortive, regardless of the degree of accuracy attained. But to take such a position was, as Poincaré well understood, theoretically equivalent to affirming a general physical law.

At first Poincaré simply defined this law without naming it:

"Experiment has revealed a multitude of facts which can be summed up in the following statement: it is impossible to detect the absolute motion of matter, or rather the relative motion of ponderable matter with respect to the ether; all that one can exhibit is the motion of ponderable matter with respect to ponderable matter."⁷

Later he referred to it as the "Principle of Relative Motion."⁸ Finally in 1904 he gave it its present name in an address delivered before the International Congress of Arts and Sciences in St. Louis. At that time he listed it as one of the six general principles of physics and defined it as:

"The principle of relativity, according to which the laws of physical phenomena should be the same, whether for an observer fixed, or for an observer carried along in a uniform movement of translation; so that we have not and could not have any means of discerning whether or not we are carried along in such a motion."⁹

In this address Poincaré went on to discuss at some length the status of the principle of relativity, noting ironically that while most theorists would have been prepared to see the principle

national de Physique réuni à Paris en 1900 (Gauthier-Villars, Paris, 1900), Vol. 1, pp. 22–23.

⁶ H. Poincaré, *Electricité et Optique* (Carré et Naud, Paris, 1901), p. 536.

⁷ Reference 4, p. 14.

⁸ H. Poincaré, *Arch. Néerl.* [2], 5, 271 (1900).

⁹ H. Poincaré, *Bull. Sci. Math.* [2], 28, 306 (1904), as reprinted in English translation in *Monist* 15, 5 (1905).

disconfirmed, the experiments themselves stubbornly confirmed its validity. He took the occasion to restate his disapproval of the way Lorentz managed to explain these results by "piling up hypotheses." The only such hypothesis for which Poincaré showed any enthusiasm was Lorentz's "ingenious idea of local time," which he examined in detail.¹⁰ Because of the important way in which this concept reappeared in Einstein's paper we summarize its evolution briefly.

In 1895 as a means of simplifying the mathematical study of electromagnetic processes in a reference system, with velocity v (in the direction of the x axis) relative to the ether, Lorentz showed the formal advantages of introducing what he termed "local time" t' differing from the "true time" t by the amount vx'/c^2 for each point on the x' axis.¹¹ At first this substitution was thought to be little more than a convenient mathematical device, but the history of science provides numerous examples of how short cuts of this kind turn out to be extremely fruitful theoretically.

Five years later Poincaré showed how Lorentz's concept of local time could be given a very simple physical interpretation, by means of an imaginary operation in which observers stationed at various points along the x' axis of the moving system synchronize their respective clocks by exchanging optical signals with an observer stationed at the origin.¹² Assuming that the velocity of light is independent of the motion of its source, the difference between true time and local time would be accounted for by an observer at rest relative to the ether as the amount each station's clock had been thrown out of "true synchrony" by its translation during the exchange of signals.

In his St. Louis address Poincaré reintroduced this interpretation of local time and explained its connection with the principle of relativity. While the time of a moving reference system (as marked by clocks synchronized in this way) would differ from the true time of clocks similarly synchronized in a fixed system, this difference would not lead to contradictions. For, in accordance with the principle of relativity all phenomena in the moving system would remain in harmony with the optical signals. Thus an observer there would

have no physical means of detecting the fact that its clocks were out of synchrony and establishing thereby his absolute motion.¹³ It is pertinent to note that all that one would have to do to bring the foregoing into accord with Einstein's general definition of time as it appears in his 1905 paper¹⁴ would be to eliminate Poincaré's nonrelativistic references to "fixed" and "moving" systems, which reveal his retention of the ether as a physically meaningful concept.

RELATIVITY AND THE ETHER

Although Poincaré was the first to advocate, in connection with the theoretical crisis in electrodynamics, that mechanical relativity should be generalized into a universal principle embracing all physical laws, the fact remains that he did not construct a *theory* of relativity. In hindsight, all the fundamental concepts seem to have lain close at hand, and the form in which Einstein was to develop such a theory was plainly in accord with Poincaré's own philosophy of science and with his views about the nature of scientific explanation. Poincaré had stressed the importance of reasoning from physical principles and had questioned the value of what he called "indifferent hypotheses," as contrasted with general mathematical expressions relating observables.¹⁵ And he was clearly searching for a better theory.

Obviously any attempt to explain why a scientist may fail to take some new step, as fundamental as the adoption of a completely different point of view, is highly questionable *per se*. At the same time it may be instructive to point out some of the ideas to which Poincaré seems to have adhered in connection with the principle of relativity, which may have kept him from giving it a more constructive theoretical role.

From our vantage point Poincaré's introduction of the principle of relativity appears to have been a transitional stage between traditional electrodynamics on the one hand and the fully relativistic theory published by Einstein on the other. Whether Einstein himself passed through such a stage in arriving at his own more radical point of view is probably unanswerable. In any

¹⁰ Reference 9, p. 10.

¹¹ Reference 3, p. 49.

¹² Reference 8, p. 272.

¹³ Reference 9, p. 10.

¹⁴ Reference 1, pp. 39-40.

¹⁵ Reference 5, pp. 8-10.

case, Poincaré's adoption of the principle of relativity seems now to have been provisional or incomplete in three respects.

First, although Poincaré was ready to postulate the exact validity of the principle with respect to all physical laws, he was troubled by the possible exception presented by gravitational phenomena.¹⁶ These were, of course, outside the scope of current experimentation on the earth's motion. If, as Laplace had concluded, gravitational effects were propagated at a speed a million times greater than the speed of light, then it could be shown that discrepancies would arise in respect to local time; that is, a system of clocks synchronized by optical signals in a moving reference system. Such discrepancies would provide an observer in that system with evidence of his absolute motion and thus disconfirm the principle of relativity. It is interesting to note that in subsequent papers on relativity Poincaré demonstrated that despite Laplace's calculation one could safely infer that both gravitational effects and light are propagated at the same velocity.¹⁷

The second respect in which Poincaré's acceptance of the principle of relativity seems provisional as compared with Einstein's lay in his belief that it might itself be explainable by a suitable revision of current electrodynamics.¹⁸ The spirit in which he had first introduced the principle was essentially a critical one arising from his objection to the number of separate hypotheses Lorentz and others were introducing to explain the various failures to detect the motion of the earth relative to the ether. The logic of his criticism was that inasmuch as all the experimental results could be summed up in one law (viz., the undetectability of absolute motion) it was reasonable to expect that a single hypothesis would account for them all. In other words, the principle of relativity remained something to be explained (e.g., by a suitable theory of the electrical constitution of matter and its interaction with the ether). Its possible theoretical role as an unexplained postulate does not seem to have been recognized by him fully, despite the

fact that he spoke of the application of general principles to various physical phenomena as "sufficient for our learning of them what we could reasonably hope to know."¹⁹

Accordingly, Poincaré does not seem to have been prepared to accept immediately one of the radical consequences of the principle of relativity: the elimination of the concept of the ether. Viewed now from the perspective of almost sixty years, any theory embracing both the ether and the principle of relativity appears to be an inevitably transitional stage. For it now seems obviously unsatisfactory to assume on the one hand that all reference systems (in uniform translational motion) are physically equivalent and at the same time to make a definite theoretical distinction singling out one of these systems as corresponding to a state of absolute rest. For if by hypothesis the same physical laws hold true in all of these systems, it is as unnecessary for a theory of electrodynamics as for a theory of mechanics to introduce the concept of a primary reference system with coordinates representing absolute space and absolute time.

The foregoing did not dawn upon all physicists at once, however, and even Poincaré appears to have been of two minds about the problem. At one time he entertained the possibility that the ether might drop out of physical theory like the ancient caloric and electrical fluids. But he went on to point out that the ether fulfilled the important theoretical role of providing a material basis for the propagation of light and avoiding in this phenomenon what would otherwise appear to be "action at a distance."²⁰ Furthermore, as we have discussed above, Poincaré seems to have believed that the principle of relativity might be accounted for in terms of the physical interactions of the ether and moving bodies. Thus in the same address in which he proposed the principle of relativity as a basic principle of physics he proceeded to suggest a new hypothesis in connection with the electrodynamics of moving bodies to the effect that it is the ether rather than the bodies themselves which was modified, losing its property of transmitting perturbations with equal velocity in all directions.²¹ From our point of

¹⁶ Reference 9, p. 12.

¹⁷ H. Poincaré, "Sur la dynamique de l'électron," *Compt. Rend.* 140, 1507 (1905). Also see H. Poincaré, *Rend. Circ. Matem. Palermo* 21, 166-175 (1906).

¹⁸ Reference 9, p. 19.

¹⁹ Reference 9, p. 5.

²⁰ Reference 5, pp. 20-22.

²¹ Reference 9, p. 19.

view, (benefiting as we do now from Einstein's insights into the problem) hypotheses of that kind appear to be self-defeating. For however satisfactorily the principle of relativity can be explained by hypotheses about the ether, the resulting theory still remains equivalent in its physical consequences to a much simpler one in which the principle of relativity is introduced as an unexplained postulate without any references to the ether whatsoever.

THEORETICAL RECONSTRUCTION

The publication in 1905 of Einstein's paper "On the Electrodynamics of Moving Bodies"²² introduced a new and decisive treatment of the theoretical problems that had been occupying Poincaré, Lorentz, and others. Properly speaking this was the first *theory* of relativity. For Einstein accepted the principle of relativity unconditionally as a fundamental postulate for the whole of physics and showed how a more satisfactory theory of electrodynamics could be constructed if one revised one's fundamental assumptions in this way. The reservations and inconsistencies that have been pointed out in Poincaré's writings on relativity were avoided altogether in Einstein's more radical approach. For while Einstein capitalized boldly on the theoretical consequences of the principle of relativity and, like Poincaré, pointed out a variety of recognized phenomena which supported it, he nowhere attempted to account for the principle itself in terms of other physical hypotheses. In these respects Einstein's scientific strategy may be compared to Newton's in the latter's theory of universal gravitation.²³ At the same time this fundamental status given to the principle of relativity as an unexplained postulate inevitably made Einstein's theory something of a stumbling stone for a generation of physicists who were intellectually committed to explaining all electrodynamic phenomena (including relativistic effects) in terms of the ether and for whom the discarding of this concept was a very high price to pay for theoretical simplicity.²⁴

²² Reference 1.

²³ See G. Holton, "On the Origins of the Special Theory of Relativity," *Am. J. Phys.* 28, 627 (1960).

²⁴ H. A. Lorentz, for example, remarked that it was a matter of individual choice whether one believed in a primary reference system (unidentifiable because of the principle of relativity) or followed Einstein and Minkowski

Although the contents of Einstein's 1905 paper are well-known we shall review its argument briefly in order to clarify the way in which Einstein attained a new point of view. After an introductory summary of the empirical grounds for the principle of relativity Einstein proposed to treat it as a postulate. The acceptance of this principle together with the principle of the constancy of the velocity of light *in vacuo* (independent of the motion of its source) permitted one to construct "a simple and consistent theory of the electrodynamics of moving bodies based on Maxwell's theory for stationary bodies."²⁵

In keeping with his promise to base his theory on the kinematics of the rigid body, Einstein proceeded to undertake a revision of traditional kinematics to bring it into conformity with his two principles. This required a preliminary criticism and redefinition of the fundamental concepts of time and length which had hitherto been employed in physical theory with a tacit acceptance of their absolute character and more or less exempted from any specific physical definition. Einstein showed how a physically meaningful concept of time could be defined operationally within a given inertial reference system by using optical signals to synchronize clocks stationed at various points within the system and by stipulating (as part of such a definition) that between any two points *A* and *B* an optical signal requires an equal time to go from *A* to *B* as from *B* to *A*. It followed immediately that the simultaneity of two events was a relative concept, for two clocks synchronized by this procedure in one inertial reference system would not be in correct synchrony from the standpoint of another inertial reference system in motion relative to the first. In the foregoing, Einstein transformed Lorentz's concept of "local time" (operationally defined in the same way Poincaré suggested) into a standard definition of time for the whole of physics, without any reference to the "true time" or "true simultaneity" which one might imagine to obtain within an "absolutely stationary" reference system.

in denying the existence of the ether altogether. He confessed his own reluctance to give up the idea of a "true time." See H. A. Lorentz, "Das Relativitätsprinzip und seine Anwendung auf einige besondere physikalische Erscheinungen" *Physik. Z.* 11, 1234 (1910).

²⁵ Reference 1, p. 38.

Einstein went on to introduce appropriate physical procedures for defining the length of a rigid body in inertial reference systems with respect to which it was at rest and in uniform relative motion, respectively. The fact that the second definition necessarily differed from the first left open the possibility that the concept of length might also prove to be a relative one.

Having completed this preliminary examination and operational definition of the concepts of time and length, Einstein was in a position to derive the transformation equations connecting the space and time coordinates established in one inertial reference system with those belonging to another reference system in uniform translational motion relative to the first. He determined these equations by a simple mathematical analysis of the way space and time coordinates would be established in the two systems by the physical procedures already defined and subject to the two principles of the theory. He proceeded to show how the transformation equations implied the reciprocal contraction of bodies in relative motion and the retardation of moving clocks, and completed the Kinematical part of his paper by deriving the relativistic formula for the addition of velocities.

Inasmuch as the new kinematics entailed by the joint validity of his two principles was expressed in the transformation equations, the second or Electrodynamical part of Einstein's paper consisted in showing how the requirement of invariance of physical laws with respect to that transformation permitted one to derive the electrodynamics of moving bodies directly from the electrodynamics of the stationary body. In the various sections of this part, Einstein demonstrated how existing laws had to be corrected or reinterpreted and how the whole theory of electrodynamics could be unified and completed by this approach.

In considering the original aspects of Einstein's theory, one must note that, unknown to him, the transformation equations deduced in the Kinematical part of his paper had appeared in almost identical form the year before in a paper by Lorentz.²⁶ Responsive to Poincaré's criticisms

²⁶ H. A. Lorentz, "Electromagnetic phenomena in a system moving with any velocity less than that of light," *Proc. Amst. Acad.* 6, 809 (1904), as reprinted in H. A.

about the number of special hypotheses he had been introducing, Lorentz undertook to give the principle of relativity a more important role in a revised theory of electrodynamics for moving bodies. His acceptance of the principle seems somewhat half-hearted now but it was clearly a step forward:

"Poincaré has objected to the existing theory of electric and optical phenomena in moving bodies that, in order to explain Michelson's negative result, the introduction of a new hypothesis has been required, and that the same necessity may occur each time new facts will be brought to light. Surely this course of inventing special hypotheses for each new experimental result is somewhat artificial. It would be more satisfactory if it were possible to show by means of certain fundamental assumptions and without neglecting terms of one order of magnitude or another, that many electromagnetic actions are entirely independent of the motion of the system."²⁷

In 1895 Lorentz had introduced the concept of "local time" together with a set of four transformation equations defining the space and (local) time coordinates in a reference system in motion relative to the ether and had shown that Maxwell's equations for the ether could be expressed identically in terms of these relative coordinates if one neglected discrepancies smaller than the first order of v/c .²⁸ Now in a manner which he later described as a "groping" one, Lorentz revised these transformation equations with the aim of maintaining the exact expression of Maxwell's equations in terms of the new relative coordinates. In this program, however, he continued to attach meaning to the concept of absolute motion relative to the ether, and while he believed that the contraction of moving bodies in the direction of such motion (as implied by the transformation) was a real physical effect he viewed the local time coordinates as essentially a mathematical artifice.²⁹ Furthermore, Lorentz failed to state the transformation equation for the time coordinates in its exact form. There-

Lorentz, A. Einstein, *et al.*, *The Principle of Relativity* (Methuen and Company Ltd., London, 1923), p. 14.

²⁷ Reference 26, p. 13.

²⁸ Reference 3, pp. 82-114.

²⁹ See H. A. Lorentz, "Deux mémoires de Henri Poincaré sur la physique mathématique," *Acta Math.* 38, 297 (1915).

fore, the transformation, as he proposed it, did not establish the complete equivalence of the "moving" and the "stationary" systems.³⁰

Einstein, on the other hand, concentrated on the physical significance of Lorentz's concept of local time and understood (as Poincaré obviously did, too) that the principle of relativity ensured the full validity of the space and time coordinates in any inertial reference system provided these had been established operationally by appropriate physical processes. In Einstein's kinematics the physical meaning of the Lorentz transformation was kept clear at each step in its derivation. Secondly, the simple logical structure of Einstein's theory made it possible for him to derive the transformation equations in their exact form by a comparatively simple argument based on his two principles. And once these equations were obtained correctly, the symmetry of their physical consequences, with respect to any two inertial reference systems, strongly justified the transition to a fully "relativistic" theory—that is, one in which all inertial reference systems were on an equal footing, the concept of the ether being discarded as "superfluous."

Actually, the whole Kinematical part of Einstein's paper could be rewritten in terms of the ether theory with surprisingly few changes. Einstein, himself, for the sake of verbal clarity referred to a "stationary" and a "moving" system, but he was careful to place such expressions within quotation marks to indicate that the distinction had no physical significance. Were he to have made a theoretical distinction between a fixed primary system and a system in absolute motion, then it would have been appropriate to eliminate the quotation marks and the whole derivation of the Lorentz transformation would have assumed a quite different meaning. In that case the transformation equations (deduced by a virtually identical argument) would have been interpreted as connecting the space and time coordinates of a system at rest in the ether with the relative coordinates of an absolutely moving system. In such an interpretation the operationally defined simultaneity would have been "true" in the fundamental system and only "apparent"

in the moving system. The physical implications of the Lorentz transformation would require a complex of similar distinctions. For example, clocks in the moving system would "really" run slow, while a clock in the fundamental system compared with a sequence of clocks in the moving system would only "apparently" run slow by the same factor. Similarly one would have to make a theoretical distinction between the "real" and "apparent" contractions of a rigid body, and so forth.

The artificiality of attempting to maintain these distinctions in physical theory when in principle there was no physical criterion for singling out the "fixed" system would surely have led soon to a simplified approach in which only relative motions were introduced and all such distinctions between "real" and "apparent" relativistic effects abandoned. But it was Einstein's unique achievement to be the first to take that revolutionary step. Theoretically it was one of profound importance, for as J. Willard Gibbs asserted: "one of the principal objects of theoretical research in any department of knowledge is to find the point of view from which the subject appears in its greatest simplicity."³¹ Students of the Special Theory of Relativity who do not appreciate the advantages of Einstein's theoretical reconstruction might profit by comparing his treatment of the electrodynamics of moving bodies with an earlier interpretation of some of the same laws in terms of the ether theory.³²

In short, Einstein's paper represented the most powerful argument yet introduced in favor of the universal validity of the principle of relativity in all branches of physics, and as such it suggested a number of important new investigations for both theory and experiment. Its creation of a modified theory of space and time led directly to Minkowski's mathematical reinterpretation of relativistic kinematics in terms of four-dimensional space-time, an innovation which, in turn, paved the way for further developments including the General Theory of Relativity.

³¹ See L. W. Wheeler, *Josiah Willard Gibbs* (Yale University Press, New Haven, Connecticut, 1951), p. 88.

³² See, for example, J. Larmor, *Aether and Matter* (Cambridge University Press, Cambridge, England, 1900). It is interesting to note that in this work Larmor anticipated Lorentz in defining the Lorentz Transformation. See p. 174.

³⁰ Henri Poincaré corrected these discrepancies the following year. See Ref. 17, p. 1505.